PROBLEM TRUSS CONNECTED TO CATENARY CABLE

The system shown in the figure is made up of a catenary cable FG supported by a truss jointed at its left end. The parameter of the catenary is equal to 6 meters and its weight per unit of length is 1 kN/m. Additionally, it is known that points F and G are at the same height, while the horizontal distance between them is 6 meters.

In the truss, A and D are pin supports. The height of points F and G is the same. The system is loaded by the set of forces listed on the figure. Every geometric measure is expressed in meters. With all this information, determine:

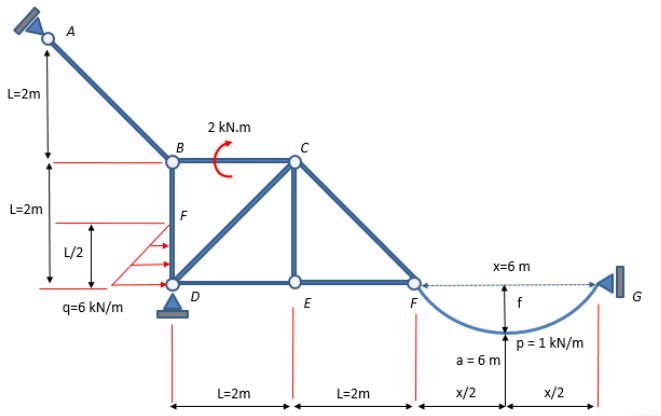
a) Maximum and minimum tension at the catenary cable.

b) Length of the catenary

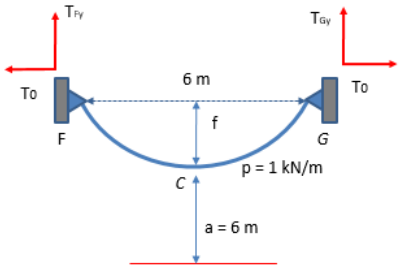
c) Internal normal forces at stretches AB, BD, CD and CF.

d) Maximum value of continuous distributed load q along stretch DF in order to avoid exceeding 1 kN.m bending moment at stretch BD.

e) Safety coefficient of the structure, considering that it is made up of a circular cross section of radius equal to 15 mm, and its material has a maximum allowable stress of 150 MPa for compressive and tensile state.



a) Reaction forces at the cable.



$$a = \frac{T_o}{p} \rightarrow T_o = 6.1 = 6\ kN$$

$$y = a\cosh\left(\frac{x}{a}\right) = 6\cosh\left(\frac{3}{6}\right) = 6.76\ m$$

$$\mathbf{T_{max}} = p.y_{max} = \mathbf{6,76\ kN}$$

$$\mathbf{T_y} = \sqrt{T_{max}^2 - T_o^2} \cong \mathbf{3,11 kN}$$

b) Length of the cable:

$$s = \sqrt{y^2 - a^2} = 3.11\ m$$

$$\mathbf{s_{tot}} = \mathbf{2.s = 6.22\ m}$$

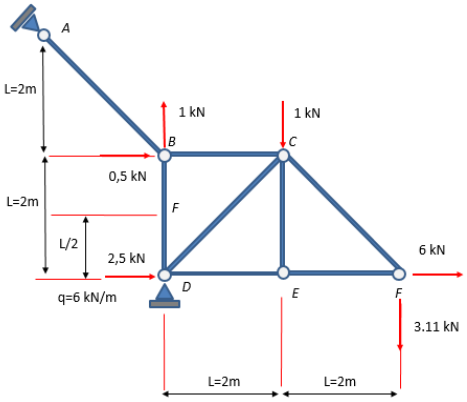
Verification of value T_y and s:

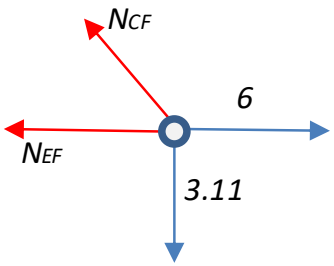
$$s = a.\sinh\left(\frac{x}{a}\right) = 6.\sinh\left(\frac{3}{6}\right) \cong 3.11\ \ m$$

$$T_y = \frac{p.s_{tot}}{2} = \frac{1\ {}^{kN}/_{m}.6.76\ m}{2} \cong 3.11\ kN$$

c) Method of the joints:

Node sequence: F→C→B



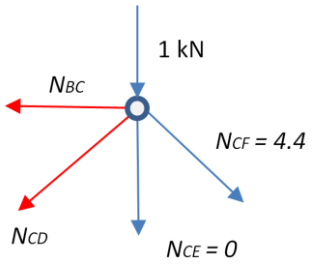


$$\sum F_y = 0 \rightarrow \frac{\sqrt{2}}{2} N_{CF} - 3{,}11 = 0 \rightarrow \mathbf{N_{CF} = 4{,}4\ kN\ (T)}$$

$$\sum F_x = 0 \rightarrow \frac{\sqrt{2}}{2} N_{CF} + N_{EF} = 6 \rightarrow N_{EF} = 2.89\ kN\ (T)$$

$$N_{EC} = 0\ kN$$

$$N_{ED} = N_{EF} = 2.89\ kN\ (T)$$

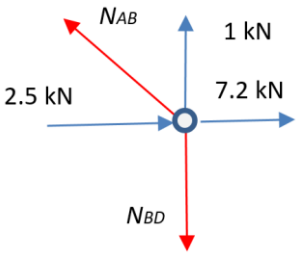


$$\sum F_y = 0 \rightarrow \frac{\sqrt{2}}{2} N_{CD} + 1 + \frac{\sqrt{2}}{2} 4.4 = 0$$

$$\mathbf{N_{CD} = 5.81\ kN\ (C)}$$

$$\sum F_x = 0 \rightarrow \frac{\sqrt{2}}{2} N_{CD} - N_{BC} + \frac{\sqrt{2}}{2} 4.4 = 0$$

$$N_{BC} = 7.21\ kN\ (C)$$

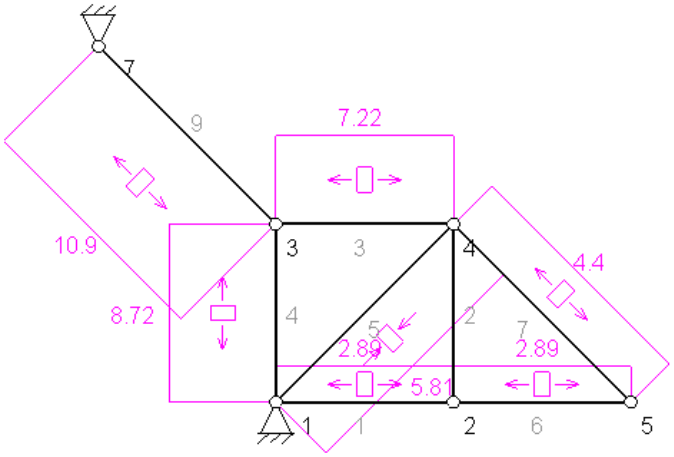


$$\sum F_x = 0 \rightarrow -\frac{\sqrt{2}}{2} N_{AB} + 7.22 + 0.5 = 0$$

$$\mathbf{N_{AB} = 10.9\ kN\ (T)}$$

$$\sum F_y = 0 \rightarrow \frac{\sqrt{2}}{2} 10.9 + 1 - N_{BD} = 0$$

$$\mathbf{N_{BD} = 8.7\ kN\ (T)}$$



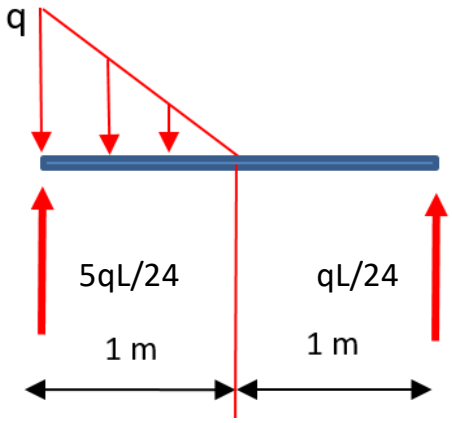
d) Maximum q at stretch BD

$$\sum M_B = 0 \rightarrow F_A . L - \frac{q}{2}\left(\frac{L}{2}\right)\left(\frac{L}{2} + \frac{2}{3}\frac{L}{2}\right) = 0$$

$$F_A = \frac{5qL}{24}$$

$$\sum F_y = 0 \rightarrow F_A + F_B = \frac{qL}{4}$$

$$F_B = \frac{qL}{24}$$



$$q(x) = q(1 - x)$$

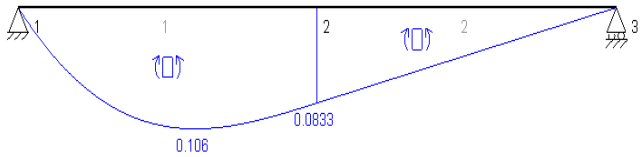
$$V_T(x) = q\left(x - \frac{x^2}{2}\right)$$

$$M_T(x) = q\left(\frac{x^2}{2} - \frac{x^3}{6}\right)$$

$$V(x) = \frac{5q}{24} - q\left(x - \frac{x^2}{2}\right) = \frac{q}{2}(x^2 - 2x + 0.833)$$

$$M(x) = \frac{5qx}{24} - q\left(\frac{x^2}{2} - \frac{x^3}{6}\right) = \frac{q}{6}(x^3 - 3x^2 + 2.5x)$$

$$M_{max} \rightarrow \frac{dM(x)}{dx} = V(x) = 0 \rightarrow x = 0.59$$



$$M_{max} = M(x = 0.59) = 0.106q = 1\ kN.m$$

$$\mathbf{q = 9.43\ kN/m}$$

e) Safety coefficient.

$$\sigma = \frac{N}{A} = \frac{10900}{\pi . R^2} = 15.42\ MPa$$

$$SC = \frac{\sigma_{max}}{\sigma_{calculated}} = \frac{150}{15.42} = \mathbf{9.72}$$